13. Gravitational Fields

13.4 Gravitational potential

Paper 4

Question Paper

1 **(a)** Define gravitational potential at a point.

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..... [2]

(b) A satellite X, of mass M, orbits a planet at a constant distance $4R$ from the centre of the planet, as shown in Fig. 1.1.

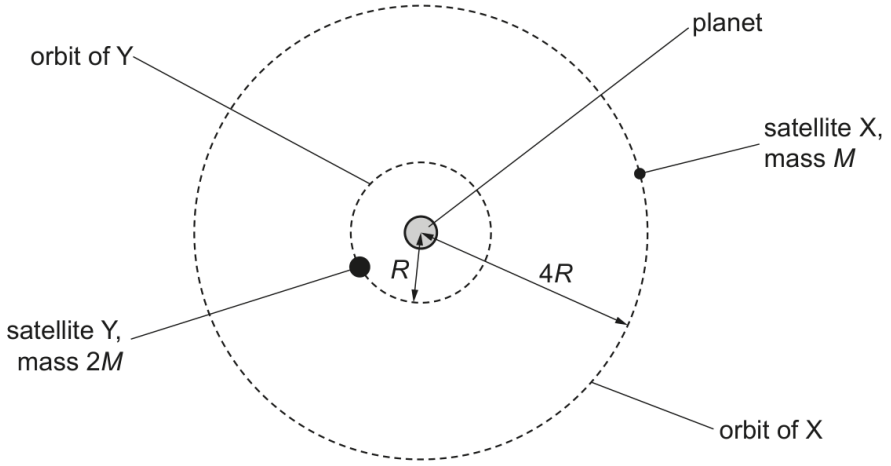


Fig. 1.1 (not to scale)

A second satellite Y, of mass $2M$, orbits the planet with orbital radius R.

The gravitational potential at X due to the planet is $-\Phi$. The planet is a uniform sphere.

(i) Explain why the gravitational potential at X is negative.

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..... [2]

(ii) State an expression, in terms of Φ, for the gravitational potential at Y due to the planet.

gravitational potential = ... [2]

2 The Sun may be considered as a uniform sphere with a mass of 1.99×10^{30} kg and a surface temperature of 5780 K.

A probe with a mass of 2.63 kg moves in a straight line towards the Sun.
When it is at a distance x from the centre of the Sun, the probe measures the gravitational field strength g due to the Sun and the radiant flux intensity F of radiation from the Sun.

(b) For the position of the probe where $x = 1.47 \times 10^{11}$ m:

(ii) determine the gravitational potential energy E_P of the probe.

E_P = ... J [2]

3 (a) (iii) State **one** similarity and **one** difference between the gravitational potential due to a point mass and the electric potential due to a point charge.

similarity: .....

.....

difference: .....

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[2]

4 **(a)** **(i)** Define gravitational potential at a point.

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..... [2]

(ii) The Moon may be considered to be an isolated uniform sphere of mass 7.3×10^{22} kg and radius 1.7×10^{6} m.

Calculate the gravitational potential at the surface of the Moon. Give a unit with your answer.

gravitational potential = ... unit ..... [2]

(b) An isolated uniform spherical planet has gravitational potential ϕ at its surface.

A particle of mass m is projected vertically upwards from the surface. The particle is given just enough kinetic energy to travel to an infinite distance away from the planet, escaping from the gravitational pull of the planet, without any additional work being done on it.

(i) Determine an expression, in terms of m and ϕ, for the gravitational potential energy E_P of the particle at the surface of the planet.

E_P = ... [1]

(ii) Show that the speed v at which the particle is projected upwards from the surface of the planet is given by

$$v = \sqrt{-2\phi}.$$

[2]

(c) A particle is moving upwards at the surface of the Moon.

Use your answer in **(a)(ii)** and the expression in **(b)(ii)** to determine the minimum speed of this particle that will result in it escaping from the gravitational pull of the Moon.

speed = ... $m\,s^{-1}$ [1]

5 **(a)** **(i)** Define gravitational potential at a point.

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..... [2]

(ii) Starting from the equation for the gravitational potential due to a point mass, show that the gravitational potential energy E_P of a point mass m at a distance r from another point mass M is given by

$$E_P = -\frac{GMm}{r}$$

where G is the gravitational constant.

[1]

(b) Fig. 1.1 shows the path of a comet of mass 2.20×10^{14} kg as it passes around a star of mass 1.99×10^{30} kg.

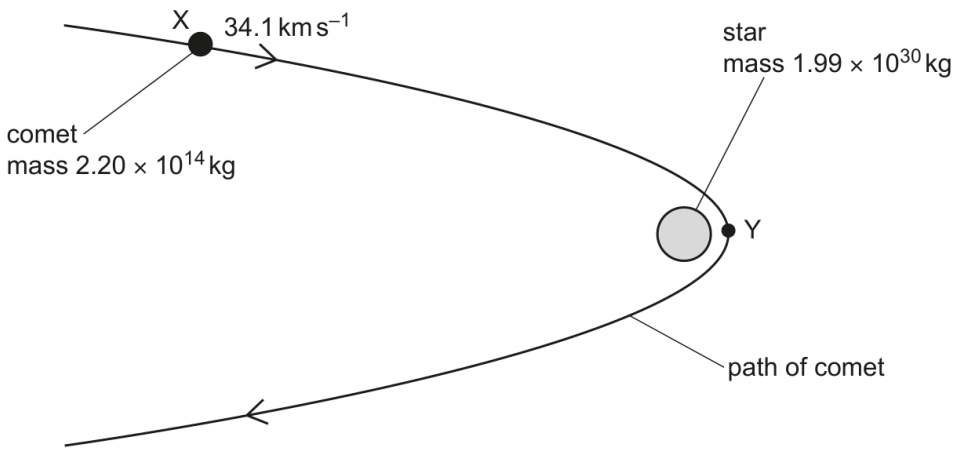


Fig. 1.1 (not to scale)

At point X, the comet is 8.44×10^{11} m from the centre of the star and is moving at a speed of $34.1\,\text{km}\,\text{s}^{-1}$.

At point Y, the comet passes its point of closest approach to the star. At this point, the comet is a distance of 6.38×10^{10} m from the centre of the star.

Both the comet and the star can be considered as point masses at their centres.

(i) Calculate the magnitude of the change in the gravitational potential energy ΔE_P of the comet as it moves from position X to position Y.

ΔE_P = ... J [2]

(ii) State, with a reason, whether the change in gravitational potential energy in **(b)(i)** is an increase or a decrease.

.....

..... [1]

(iii) Use your answer in **(b)(i)** to determine the speed, in $km\,s^{-1}$, of the comet at point Y.

speed = ... $km\,s^{-1}$ [3]

(c) A second comet passes point X with the same speed as the comet in **(b)** and travelling in the same direction. This comet is gradually losing mass. The mass of this comet when it passes point X is the same as the mass of the comet in **(b)**.

Suggest, with a reason, how the path of the second comet compares with the path shown in Fig. 1.1.

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..... [1]

6 The Earth may be assumed to be an isolated uniform sphere with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 1200 kg is in a circular orbit about the Earth in the Earth's gravitational field. The period of the orbit is 94 minutes.

(c) Rockets on the satellite are fired so that the satellite enters a different circular orbit that has a period of 150 minutes. The change in the mass of the satellite may be assumed to be negligible.

(ii) State, with a reason, whether the gravitational potential energy of the satellite increases or decreases.

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..... [1]

(iii) Determine the magnitude of the change in the gravitational potential energy of the satellite.

change in potential energy = ... J [3]

7 **(a)** Define *gravitational potential*.

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..... [2]

(b) The Earth E and the Moon M can both be considered as isolated point masses at their centres. The mass of the Earth is 5.98×10^{24} kg and the mass of the Moon is 7.35×10^{22} kg. The Earth and the Moon are separated by a distance of 3.84×10^{8} m, as shown in Fig. 2.1.

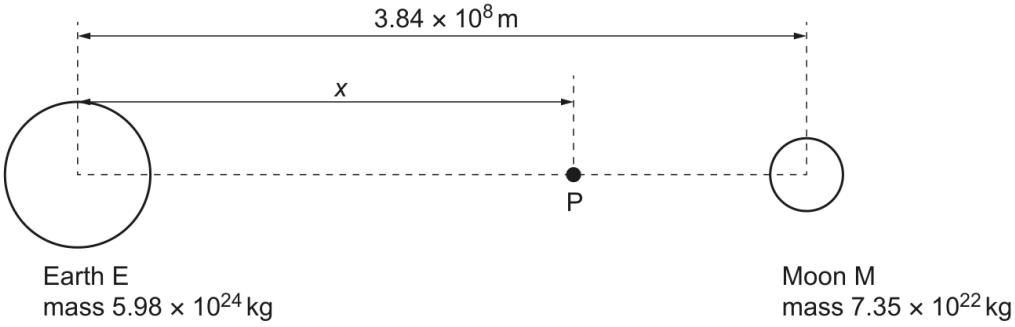


Fig. 2.1 (not to scale)

P is a point, on the line joining the centres of E and M, where the resultant gravitational field strength is zero. Point P is at a distance x from the centre of the Earth.

(iii) Calculate the gravitational potential ϕ at point P.

ϕ = ... $J\,kg^{-1}$ [3]

8 **(a)** State the relationship between gravitational potential and gravitational field strength.

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..... [2]

(b) A moon of mass M and radius R orbits a planet of mass $3M$ and radius $2R$. At a particular time, the distance between their centres is D, as shown in Fig. 2.1.

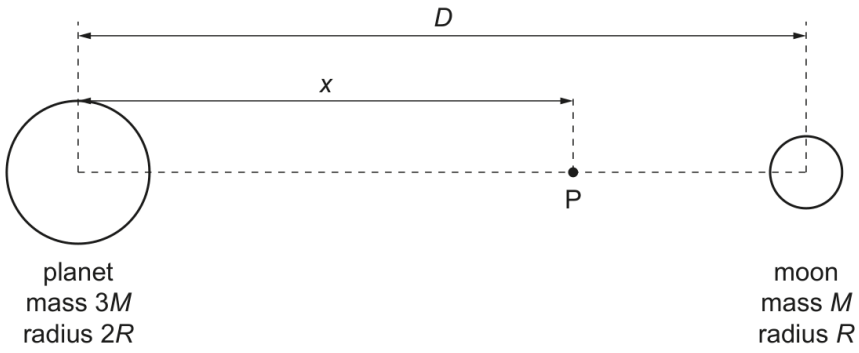


Fig. 2.1

Point P is a point along the line between the centres of the planet and the moon, at a variable distance x from the centre of the planet.

The variation with x of the gravitational potential ϕ at point P, for points between the planet and the moon, is shown in Fig. 2.2.

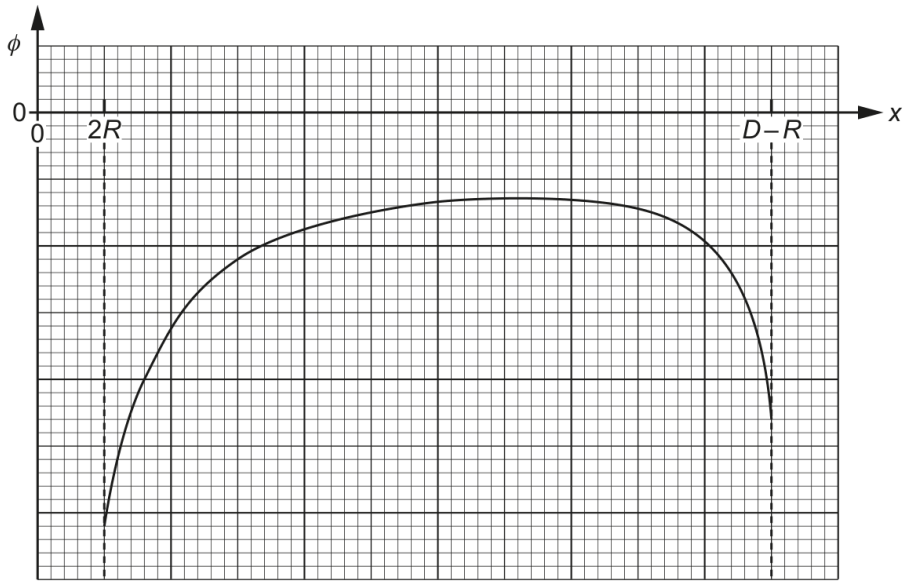


Fig. 2.2

(i) Explain why ϕ is negative throughout the entire range $x = 2R$ to $x = D - R$.

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..... [3]

(ii) One of the features of Fig. 2.2 is that ϕ is negative throughout.

Describe **two** other features of Fig. 2.2.

1. .....

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2. .....

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[2]

9 **(a)** Define *gravitational potential* at a point.

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..... [2]

(b) An isolated solid sphere of radius r may be assumed to have its mass M concentrated at its centre. The magnitude of the gravitational potential at the surface of the sphere is ϕ.

On Fig. 1.1, show the variation of the gravitational potential with distance d from the centre of the sphere for values of d from $d = r$ to $d = 4r$.

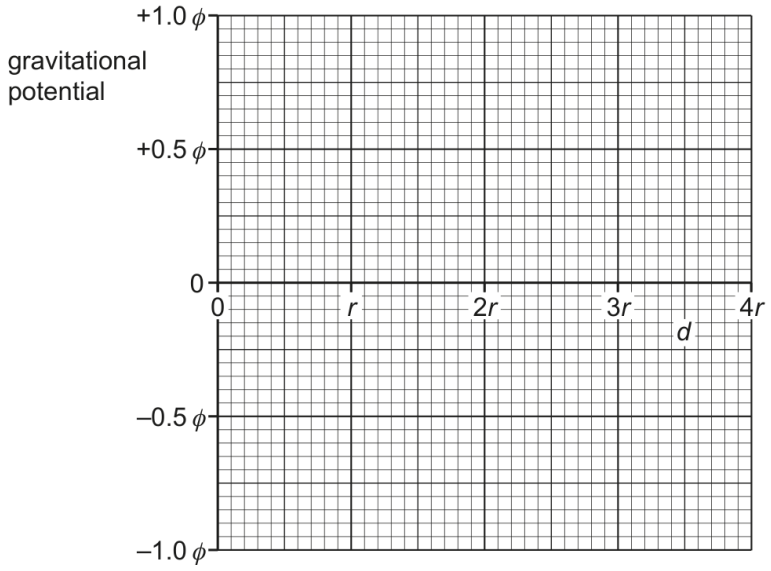


Fig. 1.1

[3]

(c) The sphere in **(b)** is a planet with radius r of 6.4×10^6 m and mass M of 6.0×10^{24} kg. The planet has no atmosphere.

A rock of mass 3.4×10^3 kg moves directly towards the planet. Its distance from the centre of the planet changes from $4r$ to $3r$.

(i) Calculate the change in gravitational potential energy of the rock.

change = ... J [3]

(ii) Explain whether the rock's speed increases, decreases or stays the same.

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..... [2]

10 **(a)** Define *gravitational potential* at a point.

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..... [2]

(b) The Earth may be considered to be a uniform sphere of radius 6.4×10^6 m with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 2.4×10^3 kg is launched from the Equator. It is placed in an equatorial orbit at a height of 5.6×10^6 m above the Earth's surface.

(i) Calculate the change ΔE_P in gravitational potential energy of the satellite for its movement from the surface of the Earth to its position in the equatorial orbit.

ΔE_P = ... J [3]

11 (b) A spacecraft is to be put into a circular orbit about a spherical planet.

The planet may be considered to be isolated in space. The mass of the planet, assumed to be concentrated at its centre, is 7.5×10^{23} kg. The radius of the planet is 3.4×10^{6} m.

(ii) One possible path of the spacecraft as it approaches the planet is shown in Fig. 1.1.

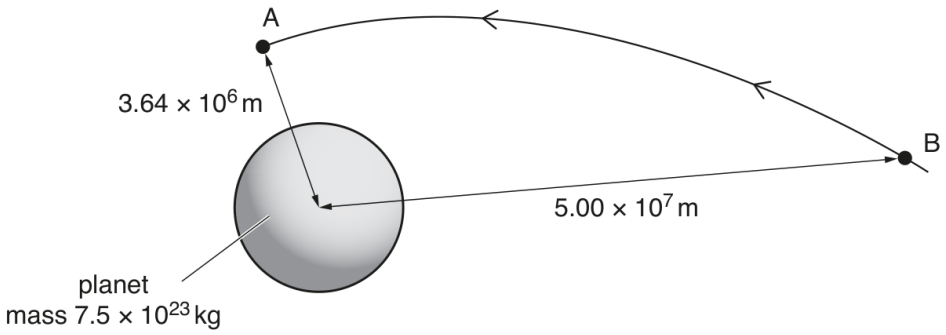


Fig. 1.1 (not to scale)

The spacecraft enters the orbit at point A with speed $3.7 \times 10^{3}\,m\,s^{-1}$.

At point B, a distance of 5.00×10^{7} m from the centre of the planet, the spacecraft has a speed of $4.1 \times 10^{3}\,m\,s^{-1}$. The mass of the spacecraft is 650 kg.

For the spacecraft moving from point B to point A, show that the change in gravitational potential energy of the spacecraft is 8.3×10^{9} J.

[3]

(c) By considering changes in gravitational potential energy and in kinetic energy of the spacecraft, determine whether the total energy of the spacecraft increases or decreases in moving from point B to point A. A numerical answer is not required.

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..... [2]

12 (b) The Moon may be considered to be a uniform sphere of diameter 3.4×10^3 km and mass 7.4×10^{22} kg. The Moon has no atmosphere.

During a collision of the Moon with a meteorite, a rock is thrown vertically up from the surface of the Moon with a speed of $2.8\,\text{km}\,\text{s}^{-1}$.

Assuming that the Moon is isolated in space, determine whether the rock will travel out into distant space or return to the Moon's surface.

[4]

13 (a) (i) State what is meant by *gravitational potential* at a point.

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(ii) Suggest why, for small changes in height near the Earth's surface, gravitational potential is approximately constant.

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.....[2]

(b) The Moon may be considered to be a uniform sphere with a diameter of 3.5×10^3 km and a mass of 7.4×10^{22} kg.

A meteor strikes the Moon and, during the collision, a rock is sent off from the surface of the Moon with an initial speed v.

Assuming that the Moon is isolated in space, determine the minimum speed of the rock such that it does not return to the Moon's surface. Explain your working.

minimum speed = ... $m\,s^{-1}$ [3]